§3 Compare the Algorithms

〖Example〗 Given (possibly negative) integers A_1, A_2, …, A_N, find the maximum value of $\sum_{k=i}^{j} A_k$.

Algorithm 1

```
int  MaxSubsequenceSum ( const int A[ ],  int  N )
{
            int  ThisSum,  MaxSum,  i,  j,  k;
/* 1*/      MaxSum = 0;   /* initialize the maximum sum */
/* 2*/      for( i = 0; i < N; i++ )  /* start from A[ i ] */
/* 3*/          for( j = i; j < N; j++ ) {   /* end at A[
/* 4*/              ThisSum = 0;
/* 5*/              for( k = i; k <= j; k++ )
/* 6*/                  ThisSum += A[ k ];  /* su                [ j ] */
/* 7*/              if ( ThisSum > MaxSum )
/* 8*/                  MaxSum = ThisSum;  /* upd  e max sum */
                } /* end for-j and for-i */
/* 9*/      return  MaxSum;
}
```

$T(N) = O(N^3)$

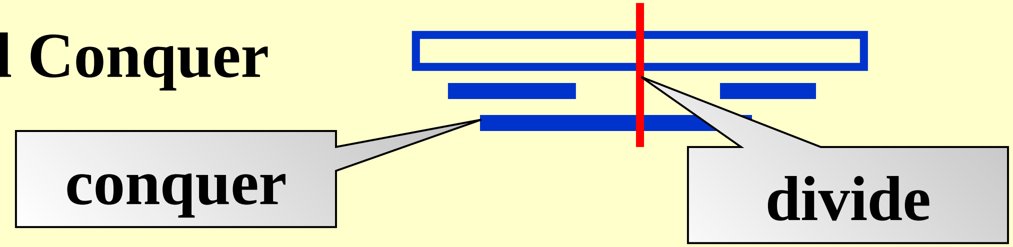
Detailed analysis is given on p.18-19.

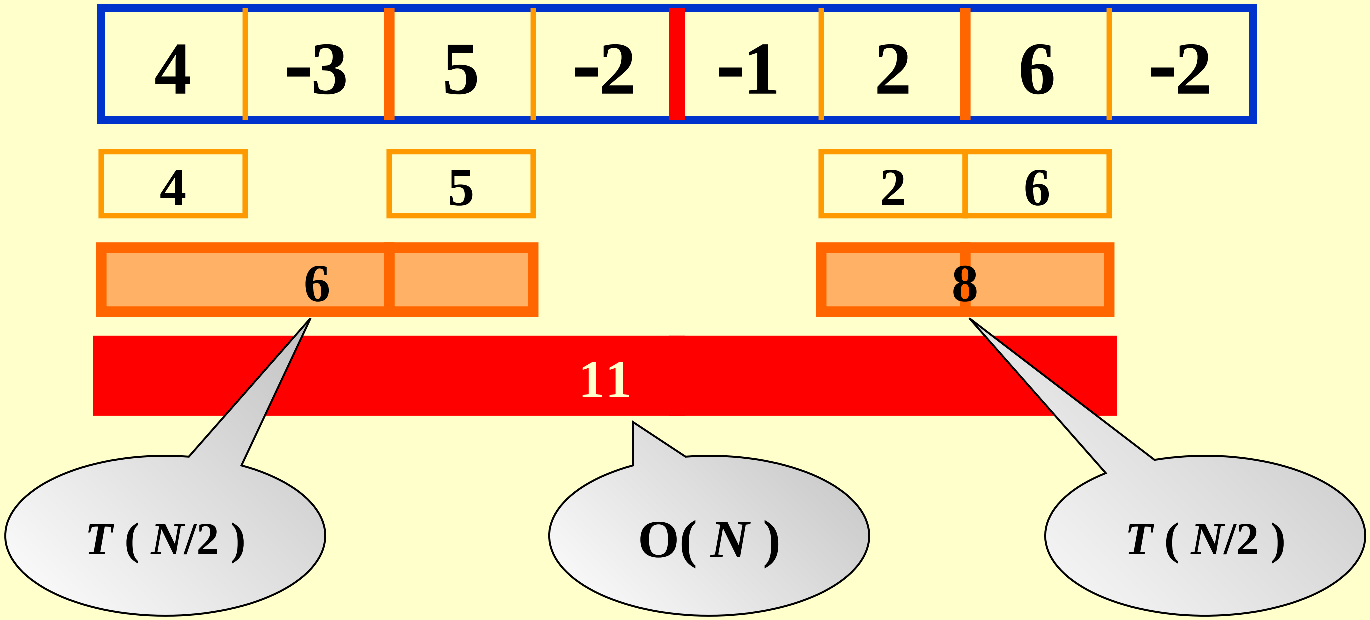
Algorithm 2

```
int  MaxSubsequenceSum ( const int A[ ],  int  N )
{
            int  ThisSum,  MaxSum,  i,  j;
/* 1*/      MaxSum = 0;   /* initialize the maximum sum */
/* 2*/      for( i = 0; i < N; i++ )  {   /* start from A[ i ] */
/* 3*/           ThisSum = 0;
/* 4*/           for( j = i; j < N; j++ ) {   /* end at A[ j ] */
/* 5*/                  ThisSum += A[ j ];  /* sum from A[ i ] to A[ j ] */
/* 6*/                  if ( ThisSum > MaxSum )
/* 7*/                       MaxSum = ThisSum;  /* update max sum */
                 }  /* end for-j */
            }  /* end for-i */
/* 8*/      return  MaxSum;
}
```

$$T(N) = O(N^2)$$

Algorithm 3 Divide and Conquer





$$T(N) = 2\,T(N/2) + c\,N, \qquad T(1) = O(1)$$

$$= 2\,[2\,T(N/2^2) + c\,N/2] + c\,N$$

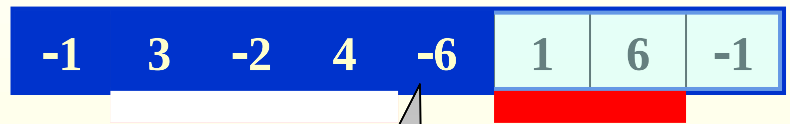
$$= 2^k\,O(1) + c\,k\,N \qquad \text{where } N/2^k$$

$$= O(N \log N)$$

The program can be found on p.21.

Algorithm 4 On-line Algorithm

```
int MaxSubsequenceSum( const int  A[ ],  int  N )
{
            int  ThisSum, MaxSum, j;
/* 1*/      ThisSum = MaxSum = 0;
/* 2*/      for ( j = 0; j < N; j++ ) {
/* 3*/           ThisSum += A[ j ];
/* 4*/           if  ( ThisSum > MaxSum )
/* 5*/                  MaxSum = ThisSum;
/* 6*/           else if ( ThisSum < 0 )
/* 7*/                  ThisSum = 0;
            }  /* end for-j */
/* 8*/      return MaxSum;
}
```



At any point in time, the algorithm can correctly give an answer to the subsequence problem for the data it has already read.

T(*N*) = O(*N*)

A[ ] is scanned once only.

Running times of several algorithms for maximum subsequence sum (in seconds)

Algorithm		1	2	3	4
Time		$O(N^3)$	$O(N^2)$	$O(N \log N)$	$O(N)$
Input Size	N =10	0.00103	0.00045	0.00066	0.00034
	N =100	0.47015	0.01112	0.00486	0.00063
	N =1,000	448.77	1.1233	0.05843	0.00333
	N =10,000	NA	111.13	0.68631	0.03042
	N =100,000	NA	NA	8.0113	0.29832

Note: The time required to read the input is not included.

§4 Logarithms in the Running Time

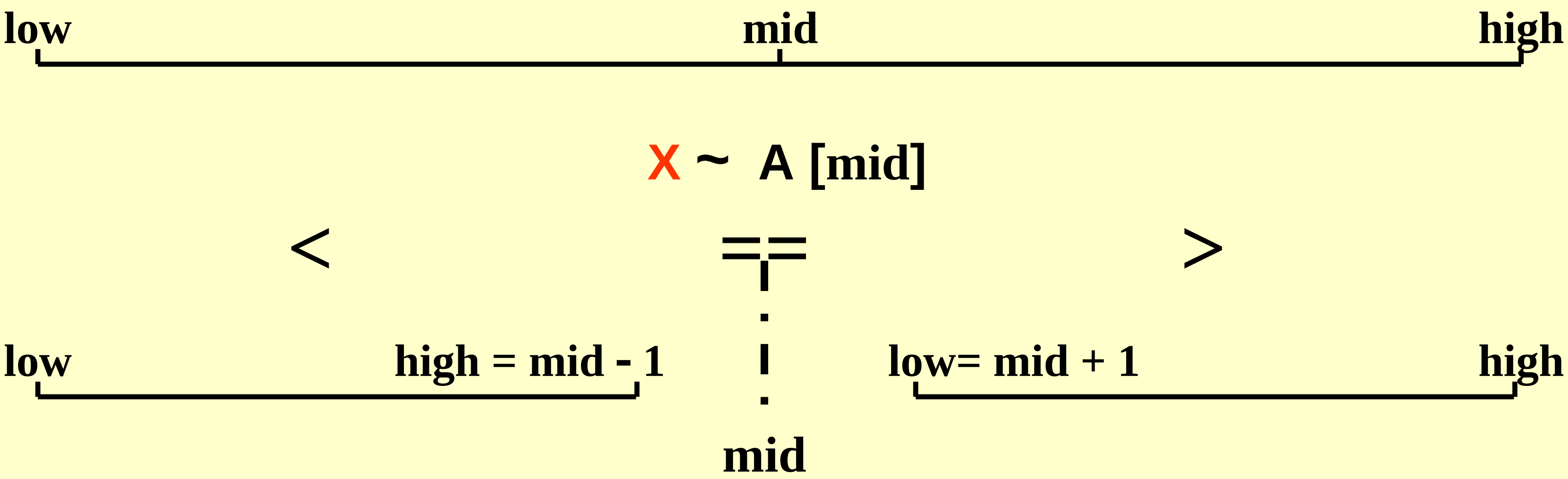
〖Example〗 **Binary Search:**

Given: A [0] ≤ A [1] ≤ …… ≤ A [N - 1] ; X

Task: Find X

Output: i if X = = A [i]

-1 if X is not found



```
int BinarySearch ( const ElementType  A[ ],
                                      ElementType  X,  int  N )
{
        int  Low, Mid, High;
/* 1*/  Low = 0;  High = N - 1;
/* 2*/  while ( 
/* 3*/
/* 4*
/*
/*
/*
/* 8*/
        }  /* end w
/* 9*/  return  NotFound; /* NotFound is defined as -1 */
}
```

$T_{worst}(N) = O(\log N)$

Very useful in

data are

Home work:

Self-study Euclid's Algorithm and Exponentiation

§5 Checking Your Analysis

Method 1

When $T(N) = O(N)$, check if $T(2N)/T(N) \approx 2$

When $T(N) = O(N^2)$, check if $T(2N)/T(N) \approx 4$

When $T(N) = O(N^3)$, check if $T(2N)/T(N) \approx 8$

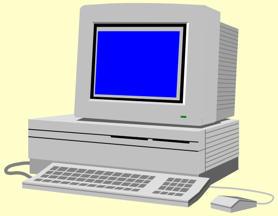
… …

Method 2

When $T(N) = O(f(N))$, check if

$$\lim_{N \to \infty} \frac{T(N)}{f(N)} \approx \text{Constant}$$

Read the example given on p.28 (Figures 2.12 & 2.13).



Laboratory Project 1

Performance Measurement

Normal: Compute X^N

Hard: Maximum Sub-matrix Sum

Due: Wednesday, October 6th, 2021 at 10:00pm

P
don't c
If it v
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and harde

I will not read and grade any program which has less than 30% lines commented.