

# §3 The Stack ADT

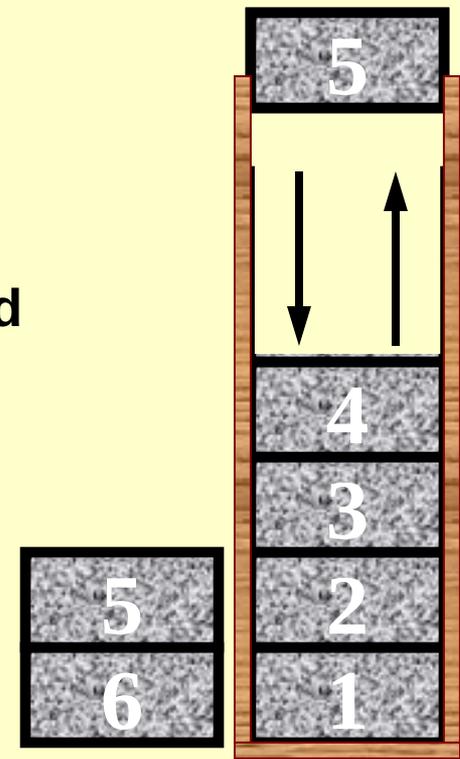
## 1. ADT

A **stack** is a Last-In-First-Out (LIFO) list, that is, an ordered list in which insertions and deletions are made at the **top** only.

**Objects:** A finite ordered list with zero or more elements.

**Operations:**

```
☞ Int IsEmpty( Stack S );  
☐ Stack CreateStack( );  
☐ DisposeStack( Stack S );  
☐ MakeEmpty( Stack S );  
☐ Push( ElementType X, Stack S );  
☐ ElementType Top( Stack S );  
☐ Pop( Stack S );
```



**Note:** A **Pop** (or **Top**) on an **empty** stack is an error in the stack ADT.

**Push** on a **full** stack is an implementation error but not an ADT error.

## 2. Implementations

### ➤ Linked List Implementation (with a header node)

👉 **Push:** ①  $\text{TmpCell} \rightarrow \text{Next} = \text{S} \rightarrow \text{Next}$

②  $\text{S} \rightarrow \text{Next} = \text{TmpCell}$

👉 **Top:** `return S->Next->Element`

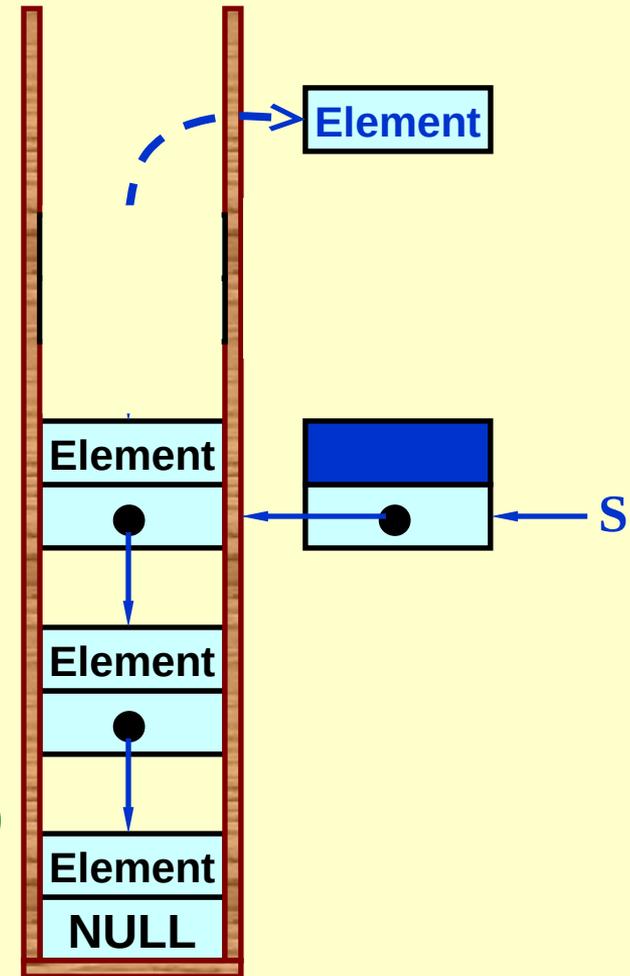
👉 **Pop:** ①  $\text{FirstCell} = \text{S} \rightarrow \text{Next}$

②  $\text{S} \rightarrow \text{Next} = \text{S} \rightarrow \text{Next} \rightarrow \text{Next}$

③ `free ( FirstCell )`



Easy! Simply keep another stack as a **recycle bin**.



## ➤ Array Implementation

```
struct StackRecord {  
    int    Capacity ;           /* size of stack */  
    int    TopOfStack;         /* the top pointer */  
    /* ++ for push, -- for pop, -1 for empty stack */  
    ElementType *Array;       /* array for stack elements */  
};
```

- Note:** ① The stack model must be well **encapsulated**. That is, no part of your code, except for the stack routines, can attempt to access the **Array** or **TopOfStack** variable.
- ② Error check must be done before **Push** or **Pop (Top)**.

Read Figures 3.38-3.52 for detailed implementations of stack operations.

### 3. Applications

#### ★ Balancing Symbols



Check if parenthesis ( ), brackets [ ], and braces { } are balanced.

```

Algorithm {
  Make an empty stack S;
  while (read in a character c) {
    if (c is an opening symbol)
      Push(c, S);
    else if (c is a closing symbol) {
      if (S is empty) { ERROR; exit; }
      else { /* stack is okay */
        if (Top(S) doesn't match c) { ERROR, exit; }
        else Pop(S);
      } /* end else-stack is okay */
    } /* end else-if-closing symbol */
  } /* end while-loop */
  if (S is not empty) ERROR;
}

```

$T(N) = O(N)$   
 where  $N$  is the length  
 of the expression.  
 This is an  
**on-line** algorithm.

# ★ Postfix Evaluation

[ Example ] An **infix** expression:  $a + b * c - d / e$

A **prefix** expression:  $- + a * b c / d e$

A **postfix** expression:  $a b c * + d e / -$

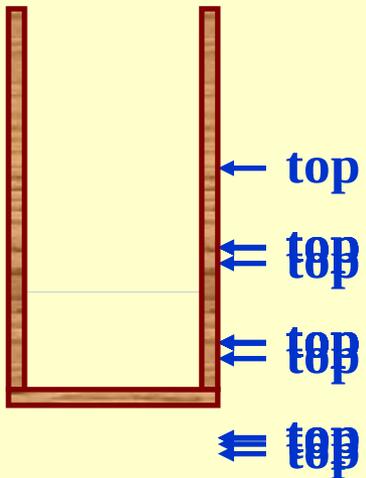
Reverse Polish notation

operand

operator with the highest precedence

operator

[ Example ]  $6 2 / 3 - 4 2 * + 8 ?$



|                           |                           |
|---------------------------|---------------------------|
| Get token: 6 ( operand )  | Get token: 2 ( operand )  |
| Get token: / ( operator ) | Get token: 3 ( operand )  |
| Get token: - ( operator ) | Get token: 4 ( operand )  |
| Get token: 2 ( operand )  | Get token: * ( operator ) |
| Get token: + ( operator ) | Pop: 8                    |

$T(N) = O(N)$ . No need to know precedence rules.

# ★ Infix to Postfix Conversion

[ Example ]  $a + b * c - d a b c * + d -$

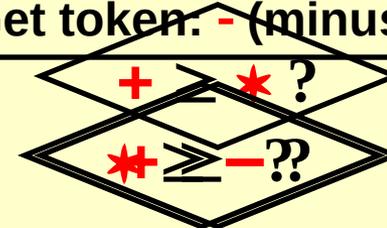
Note:

- The order of operands is the **same** in infix and postfix.
- **higher** precedence appear **before** those of lower precedence.

Isn't that simple?

Wait till you see the next example...

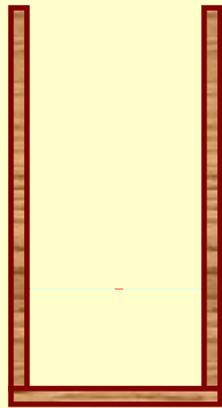
|                          |                        |
|--------------------------|------------------------|
| Get token: $a$ (operand) | Get token: $+$ (plus)  |
| Get token: $b$ (operand) | Get token: $*$ (times) |
| Get token: $c$ (operand) | Get token: $-$ (minus) |
| Get token: $d$ (operand) |                        |



[ Example ]

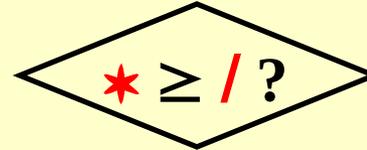
$a * ( b + c ) / c a b c + * d /$

Output:  $a b c + * d /$



← top

|                          |                          |
|--------------------------|--------------------------|
| Get token: $a$ (operand) | Get token: $*$ (times)   |
| Get token: $($ (lparen)  | Get token: $b$ (operand) |
| Get token: $+$ (plus)    | Get token: $c$ (operand) |
| Get token: $*$ (times)   | Get token: $/$ (divide)  |
| Get token: $d$ (operand) |                          |



NO!! =  $O(N)$

## Solutions:

- ① Never pop a ( from the stack except when processing a ) .
- ② Observe that when ( is **not in** the stack, its precedence is the **highest**; but when it is **in** the stack, its precedence is the **lowest**. Define **in-stack** precedence and **incoming** precedence for symbols, and each time use the corresponding precedence for comparison.

Note:  $a - b - c$  will be converted to  $a b - c -$ . However,  $2 \wedge 2 \wedge 3$  ( $2^{2^3}$ ) must be converted to  $2 2 3 \wedge \wedge$ , not  $2 2 \wedge 3 \wedge$  since exponentiation associates **right to left**.

# ★ Function Calls -- System Stack

Recursion can always be **completely removed**.  
 Non recursive programs are generally **faster** than  
 equivalent recursive programs.  
 However, recursive programs are in general  
 much **simpler and easier to understand**.

```
void PrintList ( List L )
{
  if ( L != NULL ) {
    PrintElement ( L->Element );
    PrintList( L->next );
  }
} /* a bad use of recursion */
```

```
void PrintList ( List L )
{
  top: if ( L != NULL ) {
    PrintElement ( L->Element );
    L = L->next;
    goto top; /* do NOT do this */
  }
} /* compiler removes recursion */
```

# §4 The Queue ADT

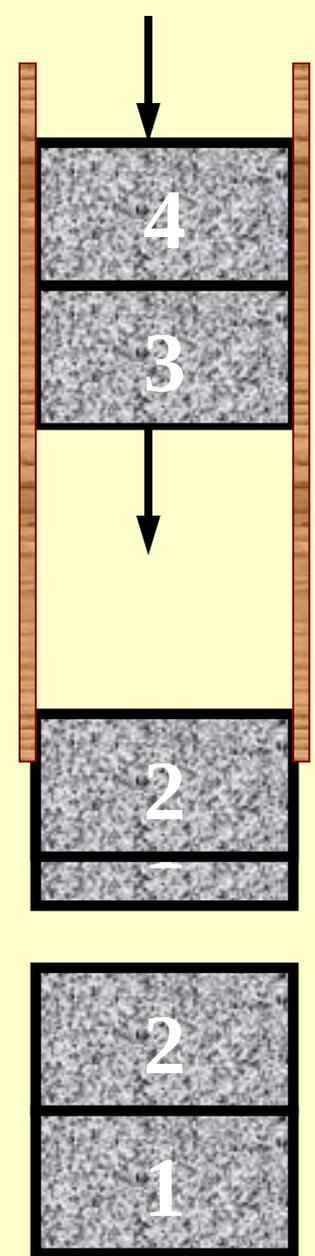
## 1. ADT

A **queue** is a First-In-First-Out (FIFO) list, that is, an ordered list in which insertions take place at one end and deletions take place at the opposite end.

**Objects:** A finite ordered list with zero or more elements.

**Operations:**

- `int IsEmpty( Queue Q );`
- `Queue CreateQueue( );`
- `DisposeQueue( Queue Q );`
- `MakeEmpty( Queue Q );`
- `Enqueue( ElementType X, Queue Q );`
- `ElementType Front( Queue Q );`
- `Dequeue( Queue Q );`



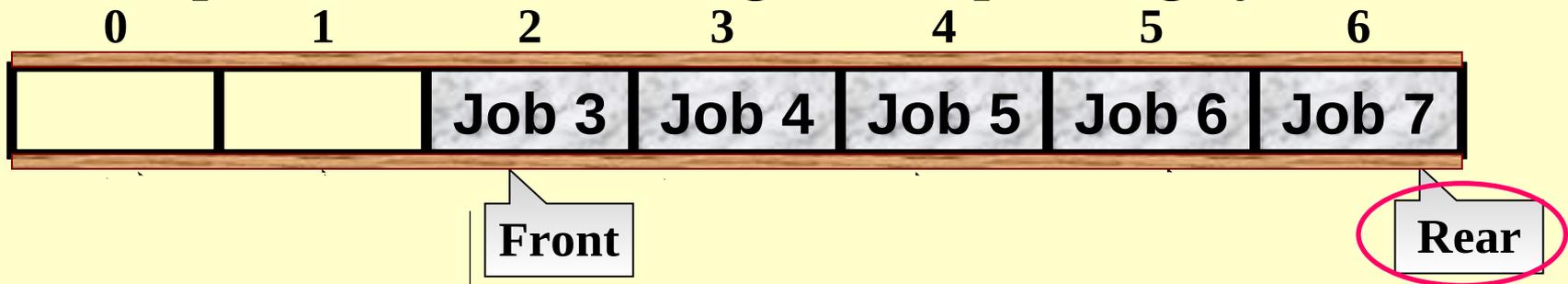
## 2. Array Implementation of Queues

(Linked list implementation is trivial)

```

struct QueueRecord {
    int    Capacity ; /* max size of queue */
    int    Front;    /* the front pointer */
    int    Rear;     /* the rear pointer */
    int    Size; /* Optional - the current size of queue */
    ElementType *Array; /* array for queue elements */
};
  
```

[ Example ] Job Scheduling in an Operating System

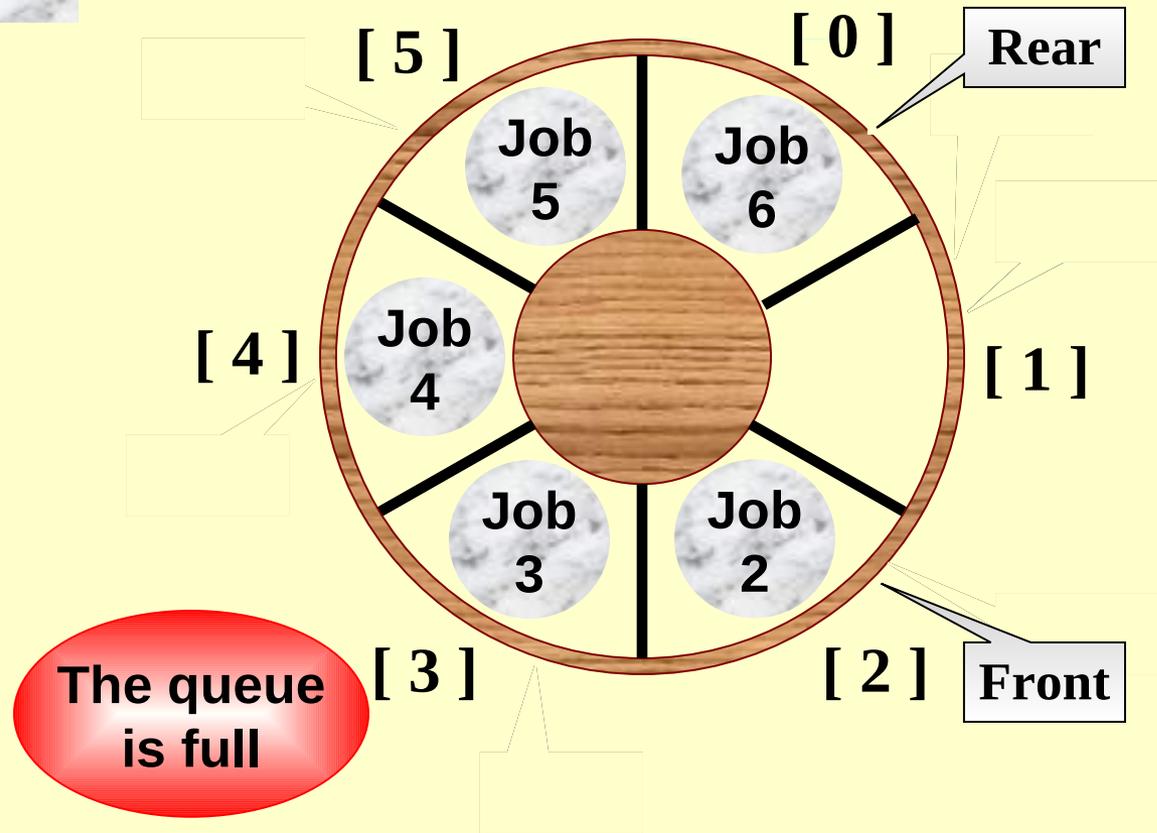


|               |               |               |               |
|---------------|---------------|---------------|---------------|
| Enqueue Job 1 | Enqueue Job 2 | Enqueue Job 3 | Dequeue Job 1 |
| Enqueue Job 4 | Enqueue Job 5 | Enqueue Job 6 | Dequeue Job 2 |
| Enqueue Job 7 | Enqueue Job 8 |               |               |

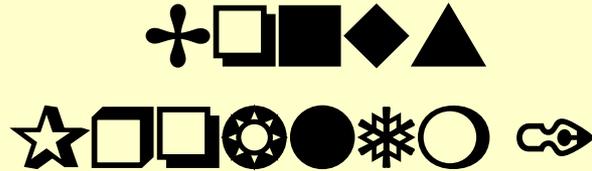
## Circular Queue:

|               |  |
|---------------|--|
| Enqueue Job 1 |  |
| Enqueue Job 2 |  |
| Enqueue Job 3 |  |
| Enqueue Job 4 |  |
| Enqueue Job 5 |  |
| Enqueue Job 6 |  |

**Question:**  
Why is the queue announced full while there is still a free space left?



**Note:** Adding a **Size** field can avoid wasting one empty space to distinguish “full” from “empty”. Do you have any other ideas?



## One Way In, Two Ways

Due: Tuesday, January 4<sup>th</sup>, 2022 at 10:00pm

(2 points)

The problem can be found and submitted at

<https://pintia.cn/>